Math Background Assignment

CS 317

Due Thursday September 4, 2008

1. Construct a truth table for $(p \Rightarrow q) \Rightarrow r$.

p	q	r	$(p \Rightarrow q) \Rightarrow r$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

2. Give a mathematical formulation of the statement "every non-negative real number has a square root" without using a square root or exponent symbol.

3. Let $P(x, y)$ denote the statement "x has eaten at y" where the domain of discourse is the set of people in town and y is the set of restaurants. Express clearly and precisely in proper English what the following mean.
   (a) $\forall y(\exists x P(x, y))$
   (b) $\exists x(\forall y P(x, y))$

4. Suppose

$$M(n) = \begin{cases} n - 10 & \text{if } n > 100 \\ M(M(n+11)) & \text{if } n \leq 100 \end{cases}$$

   Compute $M(97)$.

5. Suppose $a_n = a_{n-1} + 2n$ with $a_0 = 2$. What is a_5?

6. Find $\sum_{i=2}^{4} \sum_{j=2}^{i} j^2$.

7. In how many ways is it possible to choose 10 donuts if there are 6 different varieties?

8. How many unique strings can be formed by jumbling the letters of "`automata`"?

9. Prove by mathematical induction that $\sum_{i=1}^{n} i = n(n+1)/2$.

10. Prove that 3 divides $n^3 + 2n$ whenever n is a nonnegative integer.

11. True or false, if $n^2 - 1$ is not divisible by 3, then n is divisible by 3?

12. Prove that, for any positive integer n, $(a + b)^n \geq a^n + b^n,\ \forall a, b > 0$.

13. Prove that for $n \in \mathbb{Z}^+,\ n \geq 6,\ (n/3)^n < n! < (n/2)^n$. *Hard problem since the bounds are relatively tight (hint: use log's).*

14. Say $x_1, x_2, x_3, \ldots$ is a sequence of positive numbers satisfying $x_1 = 2$ and

$$x_{n+1} = 1 + \frac{1}{x_1 x_2 \ldots x_n}$$

for $n > 0$. Prove by contradiction that there exits a positive integer m such that $x_1 x_2 \ldots x_m > 100$.

15. (bonus) The Fibonacci numbers are defined recursively as follows:

$$x_n = \begin{cases} 1 & \text{if } n = 1 \text{ or } n = 2, \\ x_{n-1} + x_{n-2} & \text{if } n > 2. \end{cases}$$

Use induction to show that

$$x_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right].$$